QUESTIONS AND DISCUSSIONS.

EDITED BY U. G. MITCHELL, University of Kansas.

27. A certain college wishes to offer twelve hours of mathematics beyond the usual courses in analytical geometry and differential and integral calculus. Considering only the needs of students intending to specialize in pure mathematics, what courses should make up the twelve hours offered?

REPLY BY W. A. HARSHBARGER, Washburn College, Topeka, Kansas.

This subject seems at first glance to be comparatively simple and to admit of a reasonably definite answer. Every teacher of college mathematics doubtless has a pretty clear idea of what subjects a student majoring in pure mathematics should pursue in the last two years of his undergraduate course, and it is quite likely that these foot up to twice twelve hours. The real problem is one of elimination.

Introduction. Let us suppose that the student enters with the ordinary high-school preparation and the ordinary development in mathematical reasoning. The freshman year is devoted to trigonometry and so-called college algebra, three hours being devoted to each subject. The classes are large and made up of students of various grades of ability from superior to microscopic. Also they contain, side by side, students who will not pursue the subject beyond the freshman year, students who expect to take engineering work, and the few, comparatively, who will specialize in pure mathematics. Also, unless my experience is exceptional, there will be various degrees of preparation in the fundamental operations of the subject, from good to that which is really worse than none at all. Consequently the instruction in the freshman year, particularly in the algebra, must, to a considerable extent, be given to strengthening and correcting the preparation. For this reason but little college algebra can be given in the freshman year. A brief, but thorough review of the fundamental operations, surds, exponents, etc.; then a thorough treatment of the quadratic in one and two unknowns, with as full a consideration of the graph as time will permit, will occupy half the term, or probably a little less. The more advanced topics, complex numbers, permutations and combinations, determinants, theory of equations, etc., thus receive somewhat scant treatment.

In the sophomore year I suppose the student to take five hours each of analytic geometry and calculus. Here again the classes are large and of a mixed character. The work in analytic geometry will cover thoroughly the straight line, circle and conic sections, with a brief course in the geometry of three dimensions, always with the object of teaching the analytical method, rather than of teaching facts about the conics themselves.

In the calculus I presuppose a considerable facility in the processes of differentiation and integration, with simple applications and a great many problems. Infinite series, expansion by Taylor's and Maclaurin's theorems, and the definite integral may be supposed to receive careful consideration, but little will be done with partial differentiation.

We have now conducted the class up to the point contemplated by the question and probably have a comparatively small number in line for the next twelve hours in pure mathematics. As I understand the question, the purpose is to select for the next twelve hours those subjects that will best fit the student for graduate work in pure mathematics. An inventory of his work up to this time will surely show a deficiency in algebra. Hence I would have him devote the first three hours to a somewhat advanced course in algebra.

Algebra. The aim of this course is not to teach the higher algebra as that term is generally understood, but to carry the ordinary processes to a stage where their power will be appreciated; to give facility in execution; in short, to make algebra a more powerful tool in future work. My own practice is to found this course on Chrystal's *Algebra* and make numerous references to other works. It should begin at the beginning of the subject, and give a solid foundation in the fundamental laws and processes of algebra, perhaps the most difficult part of the course for the average student. A topical outline of the course is hardly necessary here, as it will be varied somewhat to meet the needs of different classes. It may well be extended to include the most essential parts of theory of equations, and the general theory of integral functions. As the object is to give facility of execution, quite as much as to teach the theory of the subject, a considerable list of select problems should be assigned, and carefully solved.

For the next three hours I would choose the subject geometry.

Geometry. This course should probably begin with a short review of the elementary course in analytic geometry, treated in a more scientific manner. From this we proceed to homogeneous point and line coördinates and the principle of duality. Here a short account of the general treatment given in courses in geometry of position, where point, line and plane, as fundamental elements lead to a multiplicity of dualistic theorems, will widen the student's horizon, and possibly awaken a desire for a later course in projective geometry. Dropping back to the point and line, the theorems on the complete quadrilateral and harmonic division are followed by the more general theorems on the descriptive properties of conics by Pascal, Chasles and others. The study of the metric properties brings in the line at infinity and the circular points at infinity. A study of projection, cross ratio and involution will complete a course sufficiently difficult for all but the best grounded classes in the subject.

For the next three hours, that is, the first term of the senior year, I would have a course in advanced calculus.

Advanced Calculus. This should begin with a brief review of the elementary course, and take up the subjects in differential calculus that were omitted or taken too briefly in the first course. Here partial differentiation should receive careful treatment. The work in infinite series, Taylor's theorem, etc., given in the first course may be considerably extended. But at least two thirds of the time should be devoted to the integral calculus. The definite integral may here be treated in a more advanced way than was possible in the first course. The student should now receive careful, though necessarily brief instruction in such

subjects as the gamma functions, elliptic integrals, line, surface and space integrals, and his knowledge of the applications of the definite integral should be considerably extended. It has been my custom to found the integral part of this course on Volume 2 of Byerly's *Calculus*, and to make liberal references to other works for both theory and application.

For the last three hours I find it difficult to decide on a subject. There are many subjects entirely suitable that would add variety to the somewhat restricted schedule given above. In many ways a course in analytic mechanics would be ideal at this time. However, keeping the graduate school in mind as the goal toward which the student is working, and recalling Dr. Bolza's rather insistent claim that a student should have two years of calculus for admission to graduate work, I venture to suggest a course in differential equations.

Differential Equations. This can be made as difficult as the ability of the class will permit, and furnishes a splendid review in the processes of integration. No outline of this need be suggested, for it will of necessity be kept quite elementary. However it should be given as many points of contact with other subjects as time will permit.

This schedule, three hours of algebra, three hours of geometry, and six hours of calculus, will probably seem to lack variety. I have kept the graduate school in mind and to the best of my ability selected subjects accordingly. For those students whose school career must stop with the bachelor's degree, or who pursue other lines of study later, a different selection may be advisable.

Conclusion. While it is not directly pertinent to the question, I venture to suggest that some students enter college prepared to begin analytic geometry, and are thus in line for eighteen hours instead of twelve. This is by no means uncommon in the college with which I am connected, owing to the good work of the mathematics department of the Topeka high school. Others will frequently desire to carry two courses simultaneously. Hence there is considerable opportunity to introduce variety where the teaching force will permit. Also a number of two hour courses can be offered, considerably widening the scope of the above schedule. In this way such subjects as history of mathematics, theory of determinants, geometry of position, geometry of three dimensions, and many others may be introduced, and the student given a much wider view without subtracting from his foundation work.

NOTES AND NEWS.

The problem department of SCHOOL SCIENCE AND MATHEMATICS is now in charge of DR. J. O. HASSLER, of the Englewood High School, Chicago.

MR. CARL GARLOUGH has been appointed to an instructorship in mathematics in Wheaton College, Wheaton, Ill.